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Preface

The papers in this volume were selected for presentation at the 16th Annual International Computing and Combinatorics Conference (COCOON 2010), held during July 19–21, 2010 in Nha Trang, Vietnam. Previous meetings of this conference were held in Singapore (2002), Big Sky (2003), Jeju Island (2004), Kunming (2005), Taipei (2006), Alberta (2007), Dalian (2008) and New York (2009).

COCOON 2010 provided a forum for researchers working in the areas of algorithms, theory of computation, computational complexity, and combinatorics related to computing. In all, 133 papers were submitted from 40 countries and regions, of which 54 were accepted. Authors of the submitted papers were from Australia (10), Bangladesh (11), Belgium (1), Canada (23), Chile (1), China (20), Colombia (1), Czech Republic (6), Denmark (1), France (25), France(1), Germany (13), Greece (2), Hong Kong (7), Hungary (2), India (18), Indonesia (8), Islamic Republic of Iran (2), Ireland (1), Israel (6), Italy (6), Japan (31), Republic of Korea(4), Malaysia (1), The Netherlands (2), New Zealand (2), Norway (3), Pakistan (1), Poland (1), Portugal (1), Russian Federation (3), Singapore (6), Slovakia (1), Spain (7), Sweden (2), Taiwan (19), Thailand (2), UK (2), USA (44), and Vietnam (15).

The submitted papers were evaluated by an international Technical Program Committee (TPC). Each paper was evaluated by at least three TPC members, with possible assistance of the external referees, as indicated by the referee list found in the proceedings. Some of these 54 accepted papers will be selected for publication in a special issue of *Algorithmica*, *Journal of Combinatorial Optimization*, and *Discrete Mathematics, Algorithms, and Application* under the standard refereeing procedure. In addition to the selected papers, the conference also included two invited presentations by Manuel Blum (Carnegie Mellon University) and Oscar H. Ibarra (University of California Santa Barbara).

We are extremely thankful to all the TPC members, each of whom reviewed about 14 papers despite a very tight schedule. The time and effort spent per TPC member were tremendous. By the same token, we profusely thank all the external referees who helped review the submissions. The TPC and external referees not only helped select a strong program for the conference, but also gave very informative feedback to the authors of all submitted papers. We also thank the local Organizing Committee for their contribution to making the conference a success. Many thanks are due to the two invited speakers and all the authors who submitted papers for consideration, all of whom contributed to the quality of COCOON 2010.

July 2010

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Table of Contents

Invited Talks

Understanding and Inductive Inference	1
<i>Manuel Blum</i>	

Computing with Cells: Membrane Systems	2
<i>Oscar H. Ibarra</i>	

Complexity and Inapproximability

Boxicity and Poset Dimension	3
<i>Abhijin Adiga, Diptendu Bhowmick, and L. Sunil Chandran</i>	

On the Hardness against Constant-Depth Linear-Size Circuits	13
<i>Chi-Jen Lu and Hsin-Lung Wu</i>	

A k -Provers Parallel Repetition Theorem for a version of No-Signaling Model	23
<i>Ricky Rosen</i>	

The Curse of Connectivity: t -Total Vertex (Edge) Cover	34
<i>Henning Fernau, Fedor V. Fomin, Geevarghese Philip, and Saket Saurabh</i>	

Counting Paths in VPA Is Complete for $\#NC^1$	44
<i>Andreas Krebs, Nutan Limaye, and Meena Mahajan</i>	

Depth-Independent Lower Bounds on the Communication Complexity of Read-Once Boolean Formulas	54
<i>Rahul Jain, Hartmut Klauck, and Shengyu Zhang</i>	

Approximation Algorithms

Multiplying Pessimistic Estimators: Deterministic Approximation of Max TSP and Maximum Triangle Packing	60
<i>Anke van Zuylen</i>	

Clustering with or without the Approximation	70
<i>Frans Schalkkamp, Michael Yu, and Anke van Zuylen</i>	

A Self-stabilizing 3-Approximation for the Maximum Leaf Spanning Tree Problem in Arbitrary Networks	80
<i>Suyaka Kamei, Hirotsugu Kakugawa, Stéphane Devismes, and Sébastien Tixeuil</i>	

Approximate Weighted Farthest Neighbors and Minimum Dilation Stars 90
John Augustine, David Eppstein, and Kevin A. Wortman

Approximated Distributed Minimum Vertex Cover Algorithms for Bounded Degree Graphs 100
Yong Zhang, Francis Y.L. Chin, and Hing-Fung Ting

Graph Theory and Algorithms

Maximum Upward Planar Subgraph of a Single-Source Embedded Digraph 110
Aimal Rextin and Patrick Healy

Triangle-Free 2-Matchings Revisited 120
Maxim Babenko, Alexey Gusakov, and Ilya Razenshteyn

The Cover Time of Deterministic Random Walks 130
Tobias Friedrich and Thomas Sauerwald

Finding Maximum Edge Bicliques in Convex Bipartite Graphs 140
Doron Nussbaum, Shuye Pu, Jörg-Rüdiger Sack, Takeaki Uno, and Hamid Zarrabi-Zadeh

A Note on Vertex Cover in Graphs with Maximum Degree 3 150
Mingyu Xiao

Computing Graph Spanners in Small Memory: Fault-Tolerance and Streaming 160
Giorgio Ausiello, Paolo G. Franciosa, Giuseppe F. Italiano, and Andrea Ribichini

Factorization of Cartesian Products of Hypergraphs 173
Alain Bretto and Yannick Silvestre

Graph Drawing and Coloring

Minimum-Segment Convex Drawings of 3-Connected Cubic Plane Graphs (Extended Abstract) 182
Sudip Biswas, Debajyoti Mondal, Rahnuma Islam Nishat, and Md. Saidur Rahman

On Three Parameters of Invisibility Graphs 192
Josef Cibulka, Jan Kynčl, Viola Mészáros, Rudolf Stolař, and Pavel Valtr

Imbalance Is Fixed Parameter Tractable 199
Daniel Lokshтанov, Neeldhara Misra, and Saket Saurabh

The Ramsey Number for a Linear Forest versus Two Identical Copies of Complete Graphs	209
<i>I W. Sudarsana, Adiwijaya, and S. Musdalifah</i>	

Computational Geometry

Optimal Binary Space Partitions in the Plane	216
<i>Mark de Berg and Amirali Khosravi</i>	
Exact and Approximation Algorithms for Geometric and Capacitated Set Cover Problems	226
<i>Piotr Berman, Marek Karpinski, and Andrzej Lingas</i>	
Effect of Corner Information in Simultaneous Placement of K Rectangles and Tableaux.....	235
<i>Shinya Anzai, Jinhee Chun, Ryosei Kasai, Matias Korman, and Takeshi Tokuyama</i>	
Detecting Areas Visited Regularly.....	244
<i>Bojan Djordjevic and Joachim Gudmundsson</i>	
Tile-Packing Tomography Is NP-hard.....	254
<i>Marek Chrobak, Christoph Dürr, Flavio Guñez, Antoni Lozano, and Nguyen Kim Thang</i>	
The Rectilinear k -Bends TSP.....	264
<i>Vladimir Estwill-Castro, Apichat Heednacram, and Francis Suraweera</i>	
Tracking a Generator by Persistence.....	278
<i>Oleksiy Busaryev, Tamal K. Dey, and Yusu Wang</i>	
Auspicious Tatami Mat Arrangements	288
<i>Alejandro Erickson, Frank Ruskey, Mark Schurch, and Jennifer Woodcock</i>	

Automata, Logic, Algebra and Number Theory

Faster Generation of Shorthand Universal Cycles for Permutations	298
<i>Alexander Holroyd, Frank Ruskey, and Aaron Williams</i>	
The Complexity of Word Circuits	308
<i>Xue Chen, Guangda Hu, and Xiaoming Sun</i>	
On the Density of Regular and Context-Free Languages.....	318
<i>Michael Hartwig</i>	
Extensions of the Minimum Cost Homomorphism Problem	328
<i>Rustem Takhonov</i>	

The Longest Almost-Increasing Subsequence 338
Amr Elmasry

Universal Test Sets for Reversible Circuits (Extended Abstract) 348
Satoshi Tayu, Shota Fukuyama, and Shuichi Ueno

Approximate Counting with a Floating-Point Counter 358
Miklós Csűrös

Network Optimization and Scheduling Algorithm

Broadcasting in Heterogeneous Tree Networks 368
Yu-Hsuan Su, Ching-Chi Lin, and D.T. Lee

Contention Resolution in Multiple-Access Channels: k -Selection in
Radio Networks 378
Antonio Fernández Anta and Miguel A. Mosteiro

Online Preemptive Scheduling with Immediate Decision or Notification
and Penalties 389
Stanley P.Y. Fung

Computational Biology and Bioinformatics

Discovering Pairwise Compatibility Graphs 399
*Muhammad Nur Yanhaona,
Md. Shamsuzzoha Bayzid, and Md. Saidur Rahman*

Near Optimal Solutions for Maximum Quasi-bicliques 409
Lusheng Wang

Fast Coupled Path Planning: From Pseudo-Polynomial to
Polynomial 419
Yunlong Liu and Xiaodong Wu

Constant Time Approximation Scheme for Largest Well Predicted
Subset 429
Bin Fu and Lusheng Wang

On Sorting Permutations by Double-Cut-and-Joins 439
Xin Chen

A Three-String Approach to the Closest String Problem 449
Zhi-Zhong Chen, Bin Ma, and Lusheng Wang

A $2k$ Kernel for the Cluster Editing Problem 459
Jianer Chen and Jie Meng

Data Structure and Sampling Theory

- On the Computation of 3D Visibility Skeletons 469
*Sylvain Lazard, Christophe Weibel, Sue Whitesides, and
 Linqiao Zhang*
- The Violation Heap: A Relaxed Fibonacci-Like Heap 479
Amr Elmasry
- Threshold Rules for Online Sample Selection 489
Eric Bach, Shuchi Chawla, and Seeun Umboh
- Heterogeneous Subset Sampling 500
*Meng-Tsung Tsai, Da-Wei Wang, Churn-Jung Liau, and
 Tsan-sheng Hsu*

Cryptography, Security, Coding and Game Theory

- Identity-Based Authenticated Asymmetric Group Key Agreement
 Protocol 510
Lei Zhang, Qianhong Wu, Bo Qin, and Josep Domingo-Ferrer
- Zero-Knowledge Argument for Simultaneous Discrete Logarithms 520
Sherman S.M. Chow, Changshe Ma, and Jian Weng
- Directed Figure Codes: Decidability Frontier 530
Michał Kolarz
- Author Index 541

The Ramsey Number for a Linear Forest versus Two Identical Copies of Complete Graphs

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Abstract. Let H be a graph with the chromatic number h and the chromatic surplus s . A connected graph G of order n is called H -good if $R(G, H) = (n - 1)(h - 1) + s$. We show that P_n is $2K_m$ -good for $n \geq 3$. Furthermore, we obtain the Ramsey number $R(L, 2K_m)$, where L is a linear forest. In addition, we also give the Ramsey number $R(L, H_m)$ which is an extension for $R(kP_n, H_m)$ proposed by Ali et al. [1], where H_m is a cocktail party graph on $2m$ vertices.

Keywords: (G, H) -free, H -good, linear forest, Ramsey number, path.

1 Introduction

Throughout this paper we consider finite undirected simple graphs. For graphs G, H where H is a subgraph of G , we define $G - H$ as the graph obtained from G by deleting the vertices of H and all edges incident to them. The order of a graph G , $|G|$, is the number of vertices of G . The minimum (maximum) degree of G is denoted by $\delta(G)$ ($\Delta(G)$). Let A be a subset of vertices of a graph G , a graph $G[A]$ represents the subgraph induced by A in G . We denote a tree on n vertices by T_n , a path on n vertices by P_n and a complete graph on n vertices by K_n . A cocktail party graph, H_n , is a graph obtained by removing n independent edges from a complete graph of order $2n$. Two identical copies of complete graphs is denoted by $2K_n$.

Given graphs G and H , a graph F is called (G, H) -free if F contains no subgraph isomorphic to G and the complement of F , \overline{F} , contains no subgraph isomorphic to H . Any (G, H) -free graph on n vertices is denoted by (G, H, n) -free. The Ramsey number $R(G, H)$ is defined as the smallest natural number n such that no (G, H, n) -free graph exists.

Ramsey Theory studies the conditions when a combinatorial object contains necessarily some smaller given object. The role of Ramsey number is to quantify

some of the general existential theorems in Ramsey theory. The Ramsey number $R(G, H)$ is called *the classical Ramsey number* if both G and H are complete graphs and in short denoted by $R(p, q)$ when $G \simeq K_p$ and $H \simeq K_q$. It is a challenging problem to find the exact values of $R(p, q)$. Until now, according to the survey in Radziszowski [10] there are only nine exact values of $R(p, q)$ which have been known, namely for $p = 3, q = 3, 4, 5, 6, 7, 8, 9$ and $p = 4, q = 4, 5$. In the relation with the theory of complexity, Burr [6] stated that for given graphs G, H and positive integer n , determining whether $R(G, H) \leq n$ holds is NP-hard. Furthermore in [11], we can find a rare natural example of a problem higher than NP-hard in the polynomial hierarchy of computational complexity theory, that is, Ramsey arrowing is \prod_2^P -complete.

Since it is very difficult to determine $R(p, q)$, one turns out to consider the problem of Ramsey numbers concerning the general graphs G and H , which are not necessarily complete, such as the Ramsey numbers for path versus path, tree versus complete graph, path versus cocktail party graph and so on. This makes the problem on the graphs Ramsey number become more interesting, especially for the union of graphs.

Let k be a positive integer and G_i be a connected graph with the vertex set V_i and the edge set E_i for $i = 1, 2, \dots, k$. *The union of graphs*, $G \simeq \bigcup_{i=1}^k G_i$, has the vertex set $V = \bigcup_{i=1}^k V_i$ and the edge set $E = \bigcup_{i=1}^k E_i$. If $G_1 \simeq G_2 \simeq \dots \simeq G_k \simeq F$, where F is an arbitrary connected graph, then we denote the union of graphs by kF . The union of graphs is called *a forest* if G_i is isomorphic to T_{n_i} for every i . In particular, if $G_i \simeq P_{n_i}$ for every i then the union of graphs is called *a linear forest*, — denoted by L .

Let H be a graph with the chromatic number h and the chromatic surplus s . The chromatic surplus of H , s , is the minimum cardinality of a color class taken over all proper h colorings of H . A connected graph G of order n is called *H -good* if $R(G, H) = (n - 1)(h - 1) + s$. In particular, for tree T_n versus complete graph K_m , Chvátal [5] showed that T_n is K_m -good with $s = 1$. Other results concerning H -good graphs with the chromatic surplus one can be found in Radziszowski [10]. Recent results on the Ramsey number for the union of graphs consisting of H -good components with $s = 1$ can be found in [2], [3], [8]. Other results concerning the Ramsey number for the union of graphs containing no H -good components can be seen in [9], [12], [13].

However, there are many graphs that have the chromatic surplus greater than one. For example, the graphs $2K_m$ and H_m have the same chromatic surplus, that is 2. Therefore, in this paper we show that P_n is $2K_m$ -good for $n \geq 3$. Based on this result, we obtain the Ramsey number $R(L, 2K_m)$. In addition, we give the Ramsey number $R(L, H_m)$ which is an extension for $R(kP_n, H_m)$ proposed by Ali et al. [1].

Let us note firstly the previous theorems and lemma used in the proof of our results.

Theorem 1. (*Gerencser and Gyarfás [7]*). $R(P_n, P_m) = n + \lfloor \frac{m}{2} \rfloor - 1$, for $n \geq m \geq 2$.

Theorem 2. (Ali et al. [1]). Let P_n be a path on n vertices and H_m be a cocktail party graph on $2m$ vertices. Then, $R(kP_n, H_m) = (n - 1)(m - 1) + (k - 1)n + 2$, for $n, m \geq 3$ and $k \geq 1$.

Lemma 1. (Bondy [4]). Let G be a graphs of order n . If $\delta(G) \geq \frac{n}{2}$ then either G is pancyclic or n is even and $G \simeq K_{\frac{n}{2}, \frac{n}{2}}$.

2 The Ramsey Goodness for P_n versus $2K_m$

The theorem in this section deals with the Ramsey goodness for a path versus two identical copies of complete graphs. First we need to prove the following two lemmas.

Lemma 2. Let t, n be positive integers and P_n be a path on $n \geq 2$ vertices and K_2 be a complete graph on 2 vertices. Then,

$$R(P_n, tK_2) = \begin{cases} n + t - 1 & \text{if } t \leq \lfloor \frac{n}{2} \rfloor, \\ 2t + \lfloor \frac{n}{2} \rfloor - 1 & \text{if } t > \lfloor \frac{n}{2} \rfloor. \end{cases}$$

Proof. We separate the proof into two cases.

Case 1. $t \leq \lfloor \frac{n}{2} \rfloor$

To prove the upper bound $R(P_n, tK_2) \leq n + t - 1$ we use induction on t . For $t = 1$, the assertion is hold from the trivial Ramsey number $R(P_n, K_2) = n$. Assume that the lemma is true for $t - 1$. We shall show that the lemma is also valid for t . Let F be an arbitrary graph on $n + t - 1$ vertices containing no P_n . We will show that \overline{F} contains tK_2 . By induction on t , \overline{F} contains $(t - 1)K_2$. Let $B = \{a_1, b_1, \dots, a_{t-1}, b_{t-1}\}$ be the vertex set of $(t - 1)K_2$ in \overline{F} , where $a_i b_i$ are the independent edges in \overline{F} for $i = 1, 2, \dots, t - 1$. By a contrary, suppose that \overline{F} contains no tK_2 . Let $A = V(F) \setminus B$, clearly $|A| = n - t + 1$. Then the subgraph $F[A]$ of F forms a K_{n-t+1} . Otherwise, if there exists two independent vertices in $F[A]$, say x and y , then the vertex set $\{x, y\} \cup B$ forms a tK_2 in \overline{F} .

Let us now consider the relation of the vertices in $F[A]$ and B . If a_i (or b_i) is not adjacent to one vertex in $F[A]$ then b_i (or a_i) must be adjacent to all other vertices in $F[A]$ since otherwise we will get two independent edges between $\{a_i, b_i\}$ and $F[A]$ in \overline{F} , together with B , the vertices form a tK_2 in \overline{F} . Without loss of generality, we may assume that each b_i is adjacent to all but at most one vertex in $F[A]$. Let us consider the subgraph $F[D]$ of F with $D = A \cup \{b_1, b_2, \dots, b_{t-1}\}$. Clearly, the subgraph $F[D]$ has n vertices and $\delta(F[D]) \geq n - t$. Since $t \leq \lfloor \frac{n}{2} \rfloor$ then $\delta(F[D]) \geq \lceil \frac{n}{2} \rceil \geq \frac{n}{2}$. Lemma 1 now applies, the subgraph $F[D]$ contains a cycle of order n . This is a contradiction because there is no P_n in F . Therefore, \overline{F} contains tK_2 .

Next it can be verified that $K_{n-1} \cup K_{t-1}$ is a (P_n, tK_2) -free graph on $n + t - 2$ vertices and hence $R(P_n, tK_2) \geq n + t - 1$.

Case 2. $t > \lfloor \frac{n}{2} \rfloor$

In this case, Theorem 1 implies that $R(P_n, tK_2) \leq R(P_n, P_{2t}) = 2t + \lfloor \frac{n}{2} \rfloor - 1$.

Conversely, $K_{\lfloor \frac{n}{2} \rfloor - 1} + \overline{K}_{2t-1}$ is a (P_n, tK_2) -free graph on $2t + \lfloor \frac{n}{2} \rfloor - 2$ vertices and therefore $R(P_n, tK_2) \geq 2t + \lfloor \frac{n}{2} \rfloor - 1$. The proof is now complete. \square

Lemma 3. *Let K_m be a complete graph on m vertices and P_3 be a path on 3 vertices. Then, $R(P_3, 2K_m) = 2m$.*

Proof. We prove the upper bound $R(P_3, 2K_m) \leq 2m$ by induction on m . We have $R(P_3, 2K_2) = 4$ from Lemma 2 and therefore the assertion holds for $m = 2$. Now assume that the assertion is true for $m - 1$, namely $R(P_3, 2K_{m-1}) \leq 2(m - 1)$. We shall show that the lemma is also valid for m . Let F be an arbitrary graph on $2m$ vertices containing no P_3 . We will show that \overline{F} contains $2K_m$. By trivial Ramsey number $R(P_2, 2K_m) = 2m$, F contains P_2 or \overline{F} contains $2K_m$. If \overline{F} contains $2K_m$ then the proof is complete. Now consider that F contains P_2 and let u and v be the two vertices of P_2 . Clearly, the subgraph $F - P_2$ of F has $2(m - 1)$ vertices. By induction hypothesis on m , the complement of $F - P_2$ contains $2K_{m-1}$. Since F contains no P_3 then the vertices u and v are not adjacent to any vertices in $2K_{m-1}$ and hence of course $\{u, v\}$ and $2K_{m-1}$ form a $2K_m$ in \overline{F} .

It is easy to verify that \overline{K}_{2m-1} is $(P_3, 2K_m)$ -free graph on $2m - 1$ vertices and hence of course $R(P_3, 2K_m) \geq 2m$. This concludes the proof. \square

Now we are ready to prove the following theorem.

Theorem 3. *Let P_n be a path of order $n \geq 3$ and K_m be a complete graph of order $m \geq 2$. Then, $R(P_n, 2K_m) = (n - 1)(m - 1) + 2$.*

Proof. Let us consider a graph $G \simeq (m - 1)K_{n-1} \cup K_1$. It can be verified that G contains no P_n and \overline{G} contains no $2K_m$. Therefore, G is $(P_n, 2K_m)$ -free graph on $(n - 1)(m - 1) + 1$ vertices and of course $R(P_n, 2K_m) \geq (n - 1)(m - 1) + 2$.

Next we prove the upper bound $R(P_n, 2K_m) \leq (n - 1)(m - 1) + 2$ by induction on $n + m$. For $m = 2$ and $n = 3$, the assertion holds from Lemmas 2 and 3, respectively. Now assume that the assertion is true for $n + m - 1$, namely

- (1). $R(P_{n-1}, 2K_m) \leq (n - 2)(m - 1) + 2$ and
- (2). $R(P_n, 2K_{m-1}) \leq (n - 1)(m - 2) + 2$.

We shall show that the theorem is also valid for $n + m$. Let F be an arbitrary graph on $(n - 1)(m - 1) + 2$ vertices. We will show that F contains P_n or \overline{F} contains $2K_m$. By induction hypothesis on n in (1), F contains P_{n-1} or \overline{F} contains $2K_m$. If \overline{F} contains $2K_m$ then it finishes the proof. Now consider that F contains P_{n-1} and let u and v be the two end vertices of the path P_{n-1} . It can be verified that the subgraph $F - P_{n-1}$ of F has $(n - 1)(m - 2) + 2$ vertices. By induction hypothesis on m in (2), the subgraph $F - P_{n-1}$ contains P_n or the complement of $F - P_{n-1}$ contains $2K_{m-1}$. If the subgraph $F - P_{n-1}$ contains P_n then the proof is done. Therefore, the complement of $F - P_{n-1}$ contains $2K_{m-1}$. If u or v is adjacent to one vertex in $2K_{m-1}$ then we have P_n in F . Conversely, if u and v are not adjacent to any vertices in $2K_{m-1}$ then we obtain $2K_m$ in \overline{F} . So, $R(P_n, 2K_m) = (n - 1)(m - 1) + 2$. The proof is done. \square

3 The Ramsey Numbers for L versus $2K_m$ or H_m

The following two theorems deal with the Ramsey numbers for the union of graphs containing H -good components with $s = 2$. In particular, for a linear forest versus $2K_m$ or H_m . First we need to prove the following lemma.

Lemma 4. *Let K_m be a complete graph on $m \geq 2$ vertices and P_n be a path on $n \geq 3$ vertices. Then, $R(kP_n, 2K_m) = (n - 1)(m - 1) + (k - 1)n + 2$, for $k \geq 1$.*

Proof. We prove the upper bound $R(kP_n, 2K_m) \leq (n - 1)(m - 1) + (k - 1)n + 2$ by induction on k . For $k = 1$, the assertion holds from Theorem 3. Assume that the assertion is true for $k - 1$, that is $R((k - 1)P_n, 2K_m) \leq (n - 1)(m - 1) + (k - 2)n + 2$. We shall show that the lemma is also valid for k . Let F be an arbitrary graph on $(n - 1)(m - 1) + (k - 1)n + 2$ vertices and suppose that \overline{F} contains no $2K_m$. We will show that F contains kP_n . By inductive hypothesis, F contains $(k - 1)P_n$. Thus, the subgraph $F - (k - 1)P_n$ of F has $(n - 1)(m - 1) + 2$ vertices. Now by Theorem 3, the subgraph $F - (k - 1)P_n$ contains P_n and hence we obtain kP_n in F .

Next construct a graph $G \simeq (m - 2)K_{n-1} \cup K_{kn-1} \cup K_1$. It is not hard to verify that G is a $(kP_n, 2K_m)$ -free graph on $(n - 1)(m - 1) + (k - 1)n + 1$ vertices and therefore $R(kP_n, 2K_m) \geq (n - 1)(m - 1) + (k - 1)n + 2$. The proof is now complete. □

Now we are ready to prove the following theorem.

Theorem 4. *For integers $k \geq 1$, let $n_k \geq n_{k-1} \geq \dots \geq n_1 \geq 3$ be integers. Let K_m be a complete graph on $m \geq 2$ vertices, P_{n_i} be a path on n_i vertices for $i = 1, 2, \dots, k$ and $L \simeq \bigcup_{i=1}^k l_i P_{n_i}$. Then,*

$$R(L, 2K_m) = \max_{1 \leq i \leq k} \left\{ (n_i - 1)(m - 2) + \sum_{j=i}^k l_j n_j + 1 \right\}, \tag{1}$$

where l_i is the number of paths of order n_i in L .

Proof. Let $t = (n_{i_0} - 1)(m - 2) + t_0 + 1$ be the maximum of the right side of the Eq. (1) achieved for i_0 , where $t_0 = \sum_{j=i_0}^k l_j n_j$. Now construct a graph $F \simeq (m - 2)K_{n_{i_0}-1} \cup K_{t_0-1} \cup K_1$. Since $n_i \geq 3$ for every $i = 1, 2, \dots, k$ then F does not contain at least one component P_{n_i} of L and hence F contains no L . Note that $\overline{F} \simeq K_{n_{i_0}-1, \dots, n_{i_0}-1, t_0-1, 1}$ is a complete m -partite graph, where the smallest partite consists of one vertex. So, there is no $2K_m$ in \overline{F} . Thus, F is a $(L, 2K_m)$ -free graph on $t - 1$ vertices and therefore $R(L, 2K_m) \geq t$.

In order to show that $R(L, 2K_m) \leq t$ we argue as follows. Let U be an arbitrary graph on t vertices and suppose that \overline{U} contains no $2K_m$. We will show that U contains L by induction on k . For $k = 1$, the theorem is true from Lemma 4. Let us assume that the theorem holds for $k - 1$. We shall show that the theorem is

also valid for k . Note that $t \geq (n_k - 1)(m - 1) + (l_k - 1)n_k + 2$. Thus, by Lemma 4, U contains $l_k P_{n_k}$. By definition of t , we obtain the following fact.

$$|U - l_k P_{n_k}| = t - l_k n_k \geq \max_{1 \leq i \leq k-1} \left\{ (n_i - 1)(m - 2) + \sum_{j=i}^{k-1} l_j n_j + 1 \right\}. \quad (2)$$

By induction hypothesis, the subgraph $U - l_k P_{n_k}$ of U contains $\bigcup_{i=1}^{k-1} l_i P_{n_i}$ and together with $l_k P_{n_k}$ we have $L \simeq \bigcup_{i=1}^k l_i P_{n_i}$ in U . Therefore, $R(L, 2K_m) = t$. This completes the proof. \square

The following theorem is an extension of Theorem 2 proposed by Ali et al. in [1].

Theorem 5. *Let $k \geq 1$ and $m \geq 3$. Let $n_k \geq n_{k-1} \geq \dots \geq n_1 \geq 3$ be integers. Let H_m be a cocktail party graph on $2m$ vertices, P_{n_i} be a path on n_i vertices for $i = 1, 2, \dots, k$ and $L \simeq \bigcup_{i=1}^k l_i P_{n_i}$. Then,*

$$R(L, H_m) = \max_{1 \leq i \leq k} \left\{ (n_i - 1)(m - 2) + \sum_{j=i}^k l_j n_j + 1 \right\}, \quad (3)$$

where l_i is the number of paths of order n_i in L .

Proof. Let $t = (n_{i_0} - 1)(m - 2) + t_0 + 1$ be the maximum of the right side of the Eq. (3) achieved for i_0 with $t_0 = \sum_{j=i_0}^k n_j$. Now we construct a graph $G \simeq (m - 2)K_{n_{i_0}-1} \cup K_{t_0-1} \cup K_1$. Since $n_i \geq 3$ for every $i = 1, 2, \dots, k$ then G does not contain at least one component P_{n_i} of L and hence G contains no L . Note that $\overline{G} \simeq K_{n_{i_0}-1, \dots, n_{i_0}-1, t_0-1, 1}$ is a complete m -partite graph which the smallest partite has one vertex. Then, \overline{G} contains no H_m . So, G is a (L, H_m) -free graph on $t - 1$ vertices and hence $R(L, H_m) \geq t$.

We will prove the upper bound $R(L, H_m) \leq t$ by induction on k . Let F be an arbitrary graph of order t and suppose that \overline{F} contains no H_m . We shall show that F contains L . From Theorem 2, we can see that the assertion is true for $k = 1$. Now let us assume that the theorem also holds for $k > 1$. Note that $t \geq (n_k - 1)(m - 1) + 2$. So, by Theorem 2, F contains $l_k P_{n_k}$. From the definition of t , we get the following fact.

$$|F - l_k P_{n_k}| = t - l_k n_k \geq \max_{1 \leq i \leq k-1} \left\{ (n_i - 1)(m - 2) + \sum_{j=i}^{k-1} l_k n_j + 1 \right\}. \quad (4)$$

By induction hypothesis, the subgraph $F - l_k P_{n_k}$ of F contains $\bigcup_{i=1}^{k-1} l_i P_{n_i}$ and hence $F \supseteq \bigcup_{i=1}^k l_i P_{n_i}$. This completes the proof. \square

4 Conclusion

To conclude this paper let us present the following two conjectures to work on.

Conjecture 1. *Let T_n be a tree on $n \geq 3$ vertices and K_m be a complete graph on $m \geq 2$ vertices. Then, $R(T_n, 2K_m) = (n - 1)(m - 1) + 2$.*

Conjecture 2. *For integers $k \geq 1$, let $n_k \geq n_{k-1} \geq \dots \geq n_1 \geq 3$ be integers. Let K_m be a complete graph on $m \geq 2$ vertices, T_{n_i} be a tree on n_i vertices for $i = 1, 2, \dots, k$ and $F \simeq \bigcup_{i=1}^k l_i T_{n_i}$. Then,*

$$R(F, 2K_m) = \max_{1 \leq i \leq k} \left\{ (n_i - 1)(m - 2) + \sum_{j=i}^k n_j + 1 \right\}, \tag{5}$$

where l_i is the number of trees of order n_i in F .

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Author Index

- Adiga, Abhijin 3
Adiwijaya 209
Anzai, Shinya 235
Augustine, John 90
Ausiello, Giorgio 160
- Babenko, Maxim 120
Bach, Eric 489
Bayzid, Md. Šhamsuzzoha 399
Berman, Piotr 226
Bhowmick, Diptendu 3
Biswas, Sudip 182
Blum, Manuel 1
Bretto, Alain 173
Busaryev, Oleksiy 278
- Chandran, L. Sunil 3
Chawla, Shuchi 489
Chen, Jianer 459
Chen, Xin 439
Chen, Xue 308
Chen, Zhi-Zhong 449
Chin, Francis Y.L. 100
Chow, Sherman S.M. 520
Chrobak, Marek 254
Chun, Jinhee 235
Cibulka, Josef 192
Csürös, Miklós 358
- de Berg, Mark 216
Devismes, Stéphane 80
Dey, Tamal K. 278
Djordjevic, Bojan 244
Domingo-Ferrer, Josep 510
Dürr, Christoph 254
- Elmasry, Amr 338, 479
Eppstein, David 90
Erickson, Alejandro 288
Estivill-Castro, Vladimir 264
- Fernández Anta, Antonio 378
Fernau, Henning 34
Fomin, Fedor V. 34
Franciosa, Paolo G. 160
- Friedrich, Tobias 130
Fu, Bin 429
Fukuyama, Shota 348
Fung, Stanley P.Y. 389
- Gudmundsson, Joachim 244
Guíñez, Flavio 254
Gusakov, Alexey 120
- Hartwig, Michael 318
Healy, Patrick 110
Heednacram, Apichat 264
Holroyd, Alexander 298
Hsu, Tsan-sheng 500
Hu, Guangda 308
- Ibarra, Oscar H. 2
Italiano, Giuseppe F. 160
- Jain, Rahul 54
- Kakugawa, Hirotosugu 80
Kamei, Sayaka 80
Karpinski, Marek 226
Kasai, Ryosei 235
Khosravi, Amirali 216
Klauck, Hartmut 54
Kolarz, Michał 530
Korman, Matias 235
Krebs, Andreas 44
Kynčl, Jan 192
- Lazard, Sylvain 469
Lee, D.T. 368
Liau, Churn-Jung 500
Limaye, Nutan 44
Lin, Ching-Chi 368
Lingas, Andrzej 226
Liu, Yunlong 419
Lokshitanov, Daniel 199
Lozano, Antoni 254
Lu, Chi-Jen 13
- Ma, Bin 449
Ma, Changshe 520

- Mahajan, Meena 44
 Meng, Jie 459
 Mészáros, Viola 192
 Misra, Neeldhara 199
 Mondal, Debajyoti 182
 Mosteiro, Miguel A. 378
 Musdalifah, S. 209

 Nishat, Rahnuma Islam 182
 Nussbaum, Doron 140

 Philip, Geevarghese 34
 Pu, Shuye 140

 Qin, Bo 510

 Rahman, Md. Saidur 182, 399
 Razenshteyn, Ilya 120
 Rextin, Aimal 110
 Ribichini, Andrea 160
 Rosen, Ricky 23
 Ruskey, Frank 288, 298

 Sack, Jörg-Rüdiger 140
 Sauerwald, Thomas 130
 Saurabh, Saket 34, 199
 Schalekamp, Frans 70
 Schurch, Mark 288
 Silvestre, Yannick 173
 Stolař, Rudolf 192
 Sudarsana, I W. 209
 Sun, Xiaoming 308
 Suraweera, Francis 264
 Su, Yu-Hsuan 368

 Takhanov, Rustem 328
 Tayu, Satoshi 348

 Thang, Nguyen Kim 254
 Ting, Hing-Fung 100
 Tixeuil, Sébastien 80
 Tokuyama, Takeshi 235
 Tsai, Meng-Tsung 500

 Ueno, Shuichi 348
 Umboh, Seeun 489
 Uno, Takeaki 140

 Valtr, Pavel 192
 van Zuylen, Anke 60, 70

 Wang, Da-Wei 500
 Wang, Lusheng 409, 429, 449
 Wang, Yusu 278
 Weibel, Christophe 469
 Weng, Jian 520
 Whitesides, Sue 469
 Williams, Aaron 298
 Woodcock, Jennifer 288
 Wortman, Kevin A. 90
 Wu, Hsin-Lung 13
 Wu, Qianhong 510
 Wu, Xiaodong 419

 Xiao, Mingyu 150

 Yanhaona, Muhammad Nur 399
 Yu, Michael 70

 Zarrabi-Zadeh, Hamid 140
 Zhang, Lei 510
 Zhang, Linqiao 469
 Zhang, Shengyu 54
 Zhang, Yong 100

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