ON THE f-COLORING OF THE CORONA PRODUCT OF K_n WITH K_m^c OR P_m

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Abstract. *An f-coloring of graph G is a generalized edge-coloring such that each vertex vin G has at most f(v) edges colored with the same color. The minimum number of colors needed to f-color G is called an f-chromatic index of G, and denoted by* $\chi_f'(G)$. *Any graph G has f -chromatic index equal to* $\Delta_f(G)$ or $\Delta_f(G)+1$, *where* $\Delta_f(G) = \max_v \{d(v)/f(v)\}$. *If* $\chi_f(G) = \Delta_f(G)$, *then G is of* $C_f 1$; *otherwise G is of* $C_f 2$. *Sufficient condition for the corona product of* K_n *with* K_m^c *or* P_m *to be of* $C_f 1$ *is given.*

Keywords*:edge coloring, f-coloring, f-chromatic index, classification, corona product*

1. Introduction

Let $G(V,E)$ be a simple graph with vertex set $V(G)$ and edge set $E(G)$. For each $v \in V(G)$, let the degree $d(v)$ of vertex v be the number of edges incident with v in the graph G and $\Delta(G)$ is the maximum degree of G. In the classical edge coloring, each vertex has at most one edge colored with the same color. Hakimi and Kariv [3] generalized the classical edge colorings and obtained many interesting results. Let f be a function which assigns a positive integer $f(v)$ to each vertex $v \in V(G)$. An f-coloring of G is a coloring of edges such that each vertex v has at most $f(v)$ edges colored with the same color. The minimum number of colors needed to f-color G is called an f-chromatic index of G, denoted by $\chi_f'(G)$. If $f(v) = 1$ for every $v \in V(G)$, the f-coloring problem is reduced to the classical edge-coloring problem.

The f-coloring problem is to find $\chi_f'(G)$ of a given graph G. It arises in many applications, including the network design problem, scheduling problem, the file transfer problem in a computer network, and so on [1,2]. The file transfer problem in a computer network modelled as follows. Assume that network . On the model, each computer v has a limit number $f(v)$ of communications ports and it takes an equal amount of time to transfer each file. Under this assumptions, the scheduling to minimize the total time for the overall transfer process corresponds to an f-coloring of a graoh G, with minimum number of colors. Note that the edges colored with the same color correspond to files that can be transferred simultaneously.

Let G be a graph,

$$\Delta_f(G) = \max_{v \in V(G)} \left\{ \left\lceil \frac{d(v)}{f(v)} \right\rceil \right\} \tag{1}$$

where $d(v)$ is the degree of vertex v in G. Hakimi and Kariv [3] gave the lower and upper bound of *f-chromatic index* of any graph G as follows :

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$$\Delta_f(G) \le \chi'_f(G) \le \max_{v \in V(G)} \left\{ \left\lceil \frac{d(v)+1}{f(v)} \right\rceil \right\} \le \Delta_f(G)+1$$

If $\chi'_f(G) = \Delta_f(G)$ then G is of $C_f\,1$ and If $\chi'_f(G) = \Delta_f(G)+1$ then G is of $C_f\,2$. Moreover, Hakimi and Kariv [1] showed that any bipartite graph is of $C_f\,1$. If G is a graph with $f(v)$ is even for each $v \in V(G)$ then G must be of $C_f\,1$. Nakano et al. in [5] gave an upper bound of *f-chromatic index* for a multigraph. Zhou et al. in [11] gave an algorithm for *f –coloring* for bipartite graphs and planar graphs.

Before we present a result on *f*-coloring, we need some preliminary concept as follows.

$$f^* = \min_{v \in V(G)} f(v) \tag{2}$$

$$V_0^*(G) = \left\{ v \mid \frac{d(v)}{f(v)} = \Delta_f(G),\ v \in V(G) \right\} \tag{3}$$

$$V^*(G) = \{ v \mid \Delta_f(G) = \left\lceil \frac{d(v)}{f(v)} \right\rceil, v \in V(G) \} \tag{4}$$

Zhang and Liu in [7] presented some sufficient conditions for a graph being $C_f\,1$ as follows.

Teorema A. [7] Let G be a graph. If $f(v^*) \nmid d(v^*)$ for every $v^* \in V^*$ then $G \in C_f\,1$.

Jiguo Yu et al. [6] studied *f-coloring* in fans and wheels. They gave classification and sufficient conditions for fans and wheels graph being $C_f\,1$. Theorem B is the result of the classificaton of fan graphs.

Teorema B. [6] Let G be a fan-graph with the core w and the path $P_n = v_1 v_2 \ldots v_n$ $(n \ge 2)$. If $n \ge 3$, then $G \in C_f\,1$.

Meanwhile, Zhang and Liu [8] found *f-chromatic index* for complete graphs and they gave a classification of complete graphs on *f-coloring*. They showed that if k and n are odd integer, $f(v) = k$ and $k \mid d(v)$ for all $v \in V$, then G is of $C_f\,2$. Otherwise G is of $C_f\,1$. In [10], Zhang et al. presented a classification of regular graphs on *f-coloring.* They gave a some sufficient condition for a regular graph being $C_f\,1$ or $C_f\,2$.

Let G dan H be two graph with n and m vertices, respectively. The *corona product* of graphs G and H, denoted by $G \odot H$, is defined as a graph obtained by taking one copy of G and n copies H (say $H_1, H_2, \ldots, H_n$), and joining the i-th vertex of G to every vertices in H_i. Let K_n be a complete graph with n vertices and K_m^c be complement of a complete graph with m vertices. Let P_m be a path with m vertices. However, characterization of all graphs in $C_f\,1$ is still not completely solved. In this paper, we study *f*-coloring on graphs $K_n \odot K_m^c$ and $K_n \odot P_m$.

2. The Main Results

We have the new results as following two theorems :

Theorem 1. For every $n,m \in \mathbb{Z}^+$ then maka $K_n \odot K_m^c \in C_f\,1$

Proof :

Let C be color set for color each edges $E\left(K_n \odot K_m^c\right)$, where $|C| = \Delta_f$.

From the definition in eq. (4), we know that $v \in V^*$, for every $v \in K_n$.

(1) For every $v \in V \setminus V^*$, satisfy :

$$\left\lceil \frac{d(v)}{f(v)} \right\rceil = 1 \le \Delta_f , \text{ because } V \setminus V^* = V(K_m^c).$$

Hence, pendant edge can be colored by color in C.

(2) $v \in V^*$, $d(v)$ is uniform. Let $f(v) = k$

Case 1. $f(v)$ Œ$d(v)$, for every v

By theorem A, it is proved

Case 2: $k \mid d(v)$, for every v

Let $E_i\left(K_i \odot K_m^c\right)$ is a edges set which union of joining the first vertex of G to every vertices in H_1, assign

we give an f-coloring of G. We assign color in C to all edges of $E_1\left(K_1 \odot K_m^c\right)$ use the first color of C for k edges, and so on. We have of dilakukan dengan mengassign k buah warna yang sama pada setiiap edge secara terurut

Case 1. $\Delta f(G) = 1$.

In this case, $d(v) \le f(v)$ for all $v \in V$. Obviously, we can f-color graph G with one color and thus G is of Cf 1.

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Case 3. $\Delta f(G) = 3$.

In this case, G is of Cf 1 if and only if $\chi'(G) = 3$.

Subcase 3.1. $d(w) = 3r(r \ge 1)$.

We draw the circle $C_n = v_1 v_2 v_3 \ldots v_n v_1$ in a clockwise direction. Starting from $v_1 v_2$, f-color the edges on the circle with the color 1, 2, and 3 alternately. Then f-color the edges wv_i $(i = 1, 2, \ldots, n)$ with the color 2, 3, and 1 alternately. Thus, a desired f-coloring of G is obtained.

The recent result, we give a sufficient conditon of corona product graphs, $K_n \odot C_m$ on f-coloring. Our results are shown in the following theorem.

In this paper, we have not completely solved the classification problem of corona of product of graphs on f-coloring. One could study the classification on f-coloring for corona product of the others graphs.

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