THE *f*-COLORING OF THE CORONA PRODUCT OF COMPLETE GRAPH WITH CYCLE GRAPH

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Abstract. *Let $G = (V, E)$ be a simple graph and f be a function which assigns a positive integer $f(v)$ to each vertex $v \in V$. An f-coloring of graph G is a generalized edge-coloring such that each vertex $v \in V$ has at most $f(v)$ edges colored with the same color. The minimum number of colors needed to f-color G is called an f-chromatic index of G, and denoted by $\chi_f'(G)$. The f–chromatic index of G is equal to $\Delta_f(G)$ or $\Delta_f(G)+1$, where $\Delta_f(G) = \max_v\{d(v)/f(v)\}$. G is called in the class-1, denoted by C_f1, if $\chi_f'(G) = \Delta_f(G)$; otherwise G is called in the class-2, denoted by C_f2. In this paper, we assume that $f(v) = k$ for every $v \in V(G)$. We showed that for every $n,m \in \mathbb{Z}^+$, the corona product of K_n with C_m to be in C_f1.*

Keywords : *edge coloring, f-coloring, f-chromatic index, classification, corona product*

1. Introduction

Let $G = (V(G), E(G))$ be a simple graph with vertex set $V(G)$ and edge set $E(G)$. For each $v \in V(G)$, let $d(v)$ denote the degree of v. In the classical edge coloring, every two adjacent edges gets two diferent colors. Hakimi and Kariv [3] generalized the classical edge coloring. Let f be a function from $V(G)$ to a set of positive integers. An f-coloring of G is a coloring of edges such that each vertex v has at most $f(v)$ edges colored with a same color. The minimum number of colors needed to f-color G is called an f-chromatic index of G, denoted by $\chi_f'(G)$. If $f(v) = 1$ for every $v \in V(G)$, the f-coloring problem is reduced to the classical edge-coloring problem.

The f-coloring problem is to find $\chi_f'(G)$ of a given graph G. It arises in many applications, including the network design problem, scheduling problem, the file transfer problem in a computer network [1,2]. The file transfer problem in a computer network is modelled as follows. Each computer is represented by vertex and every two computer in the file transfer process is represented by edge. Each computer v has a limit number $f(v)$ of communications ports and it takes an equal amount of time to transfer each file. Under this assumptions, the scheduling to minimize the total time for the overall transfer process corresponds to an f-coloring of a graph G, with minimum number of colors. Note that the edges colored with the same color correspond to files that can be transferred simultaneously.

Let G be a graph,

$$\Delta_f(G) = \max_{v \in V(G)} \left\{ \left\lceil \frac{d(v)}{f(v)} \right\rceil \right\} \tag{1}$$

where $d(v)$ is the degree of vertex v in G. Hakimi and Kariv [3] gave the lower and upper bound of *f-chromatic index* of any graph G as follows :

$$\Delta_f(G) \le c'_f(G) \le \Delta_f(G) + 1. \tag{2}$$

G is called in the *class-1*, denoted by C_f1, if $\chi_f'(G) = \Delta_f(G)$; otherwise G is called in the *class-2*, denoted by C_f2. Moreover, Hakimi and Kariv [1] showed that any bipartite graph is in C_f1. If G is a graph with $f(v)$ is even for each $v \in V(G)$, then G must be in C_f1. Nakano et al. in [5] gave an upper bound of

f-chromatic index for a multigraph. Zhou et al. in [11] gave an algorithm for *f –coloring* for bipartite graphs and planar graphs. Jiguo Yu et al. [6] studied
f-coloring in fans and wheels. They gave sufficient conditions for fans and wheels graph being $C_f 1$.

Let,

$$V_0^*(G) = \left\{ v \mid \Delta_f(G) = \frac{d(v)}{f(v)},\ v \in V(G) \right\} \tag{3}$$

$$V^*(G) = \left\{ v \mid \Delta_f(G) = \left\lceil \frac{d(v)}{f(v)} \right\rceil, v \in V(G) \right\} \tag{4}$$

Zhang and Liu in [7,8] presented some sufficient conditions for a graph being $C_f 1$ as follows.

Teorema 1. [7] Let G be a graph. If $f(v^*) \nmid d(v^*)$ for every $v^* \in V^*$, then $G \in C_f 1$.

Teorema 2. [8] Let G be a graph. If the subgraph induced by $V_0{}^*$ is forest, then $G \in C_f 1$.

Meanwhile, Zhang and Liu [8] found the *f-chromatic index* for complete graphs and they gave a classification of complete graphs on *f-coloring*. They showed that if k and n are odd integers, $f(v) = k$, and $k \mid d(v)$ for all $v \in V$, then G is of $C_f 2$. Otherwise G is of $C_f 1$. In [10], Zhang et al. presented a classification of regular graphs on *f-coloring*. They gave some sufficient conditions for a regular graph being $C_f 1$ or $C_f 2$.

Let G dan H be two graph with n and m vertices, respectively. The *corona product* of graphs G and H, denoted by $G \odot H$, is defined as a graph obtained by taking one copy of G and n copies of H (say $H_1, H_2, ..., H_n$), and joining the i-th vertex of G to every vertices in H_i. Let K_n be a complete graph with n vertices and C_m be a cycle graph with m vertices. We give In the figure 1, an example of the corona product of K_4 with C_3.

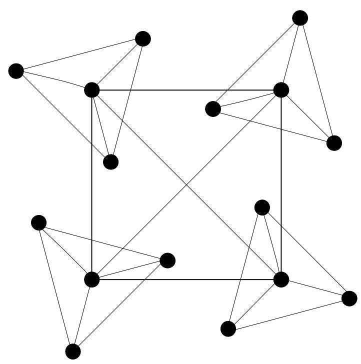

Figure 1. The graph of corona product, $K_4 \odot C_3$

However, characterization of all graphs in $C_f 1$ is still not completely solved. In this paper, we study *f*-coloring on graphs $K_n \odot C_m$.

2. The Main Results

In this section, we show that the corona product of a complete graph with a cycle graph is in $C_f 1$, if the value of f is constant for all vertices. Before that, we give some definition. Let C be a color set. An edge colored with color $c \in C$ is called an *c-edge*. The number of c-edges of G which incident with a vertex v is denoted by $d(v,c)$. Let $f(v) = k$ for every $v \in V(G)$, define $m(v,c) = k - d(v,c)$. Color c is available at vertex v if $m(v,c) \geq 1$.

Theorem 3. Let $G = K_n \odot C_m$. For every $n,m \in \mathbb{Z}^+$ and $f(v) = k$ for every $v \in V(G)$ then $G \in C_f 1$.

Proof :

Let C be a color set for f-coloring each edges $E(G)$, where $|C| = \Delta_f(G)$.

From the definition in eq. (4), we know that $v \in V^*$, for every $v \in K_n$.

(1) For every $v \in V \setminus V^*$, we have
$\left\lceil \frac{d(v)}{f(v)} \right\rceil = 1 \leq \Delta_f(G)$, because $V \setminus V^* = V(K_m^c)$.
Hence, the pendant edges can be colored by colors in C.

(2) Let $v \in V^*$, we know that $d(v) = n + m - 1$. We divide the proof into two cases :

Case 1. $k \nmid d(v)$, for every v.

By theorem 1, then $G \in C_f 1$

Case 2: $k \mid d(v)$, for every v.

Let C^* is a multiset, $C^* = \{c_i \in C \mid$ mulitiplicity of c_i is k, for every $i\}$.
It's clear that $|C^*| = k.a$, where $a = \Delta_f(G)$.
First step, we assign colors in C^* to all of edges which incident with $v_1 \in V^*$.
The color c_1 for k edges, color c_2 for another k edges, and so on. Therefore, ka colors have been used to color the all of edges which incident with $v_1 \in V^*$.
We obtained $m(v, c_i) = k - 1$, for some i and for every $v \in V^* - \{v_1\}$.
Second step, we assign colors in $C^* - \{c_i\}$ to all of edges which incident with $v_2 \in V^*$.
Now, $m(v, c_i) = k - 2$, for some i and for every $v \in V^* - \{v_1, v_2\}$.
Generally, we can assign $|C^*| - (j - 1)$ colors to all of edges which incident with $v_j \in V^*$, $1 \leq j \leq n$.
Thus, a desired f-coloring of G is obtained by Δ_f colors. $\square$

Next theorem shows that the corona product of K_n with C_m to be of $C_f 1$, where $f(v) = k$ for every $v \in K_n \odot C_m$ $(m \geq 3)$.

Theorem 4. Let $n, m \in \mathbb{Z}^+$. If $G = K_n \odot C_m$ and $f(v) = k$ for every $v \in V(G)$, then $G \in C_f 1$.

Proof :

Let C be a color set for f-coloring each edges $E(G)$, where $|C| = \Delta_f(G)$.

Case 1. $k \nmid d(v)$, for every $v \in V^*$

By theorem 1, then $G \in C_f 1$

Case 2: $k \mid d(v)$, for every $v \in V^*$

(a) When $n = 1$, we know that $G = W_n$, Then $G \in C_f 1$.
(b) When $n = 2$, by theorem 2, Then $G \in C_f 1$.
(c) When $n > 2$, let C^* is a multiset, $C^* = \{c_i \in C \mid$ mulitiplicity of c_i is k, for every $i\}$.
Assume, we erase all of edges of C_m in G. By using a similar method in the proof of Theorem 3, It is clear that we have colored the $K_n \odot K_m^c$ by Δ_f colors.
Finally, we assign colors for the paths. The edges of C_m are colored alternately with the colors which was used in $E(K_n)$.
Thus, a desired f-coloring of G is obtained by Δ_f colors. $\square$

3. Future Works

In the future, we will give a sufficient condition for a graph the corona product of K_n with another graph. Therefore, we will work on f-coloring when $f(v)$ is given for every vertex $v \in V(G)$.

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