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ON SUPER EDGE ANTI MAGIC TOTAL LABELING FOR *t*-JOINT COPIES OF WHEEL

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Abstract

A (p, q)-graph G is called (a, d)-edge anti magic total, (a, d)-EAMT if there exist integers $a > 0$, $d \geq 0$ and a bijection $\lambda : V \cup E \to \{1, 2, ..., p + q\}$ such that $W = \{w(xy) : xy \in E\} = \{a, a + d, ..., a + (q - 1)d\}$, where $w(xy) = \lambda(x) + \lambda(y) + \lambda(xy)$

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is the edge weight of xy. An (a, d)-EAMT labeling λ of G is called super, (a, d)-SEAMT if $\lambda(V) = \{1, 2, ..., p\}$. In this paper, we show that t joint copies of wheel has a $((2n+2)t+2, 1)$-SEAMT labeling for even $n \geq 4$, and $t \geq 2$.

1. Introduction

We consider finite undirected graphs without loops and multiple edges. The notation $V(G)$ and $E(G)$ stand for the vertex set and edge set of graph G, respectively. Let $e = \{u, v\}$ (in short, $e = uv$) denote an edge connecting vertices u and v in G. A graph P_n denotes a path on n vertices. Other standard terminologies and notations for graph theoretic ideas we follow the book of [5].

We denote by (p, q)-*graph* G a graph with p vertices and q edges. A (p, q)-graph G is called (a, d)-*edge anti magic total*, (a, d)-*EAMT* if there exist integers $a > 0$, $d \geq 0$ and a bijection $\lambda : V \cup E \to \{1, 2, ..., p+q\}$ such that the set of edge-weights is

$$W = \{w(xy) : xy \in E\} = \{a, a+d, ..., a+(q-1)d\},$$

where $w(xy) = \lambda(x) + \lambda(y) + \lambda(xy)$. We shall follow [7] to call $w(xy) = \lambda(x) + \lambda(y) + \lambda(xy)$ the *edge-weight* of xy, and W the *set of edge-weights* of the graph G. In particular, an (a, d)-EAMT labeling λ of a (p, q)-graph G is super if $\lambda(V) = \{1, 2, ..., p\}$. For the rest of paper, we will denote super (a, d)-EAMT of G by (a, d)-SEAMT.

For any (a, d)-SEAMT labeling on a (p, q)-graph G, the maximum edge-weight is no more than $p + (p-1) + (p+q)$. Thus, $a + (q-1)d \leq 3p + q - 1$. Similarly, the minimum possible edge-weight is at least $1 + 2 + p + 1$. This implies that $a \geq p + 4$. Therefore, we have

$$d \leq \frac{2p + q - 5}{q - 1}. \quad (1)$$

In general, for any (a, d)-EAMT labeling on a (p, q)-graph G, the maximum edge-weight is no more than $(p + q - 2) + (p + q - 1) + (p + q)$. Thus, $a + (q - 1)d \leq 3p + 3q - 3$. Similarly, the minimum possible edge-weight is at least $1 + 2 + 3$. Consequently, $a \geq 6$. So, we have

$$d \leq \frac{3p + 3q - 9}{q - 1}. \tag{2}$$

A number of classification studies on (a, d)-SEAMT (resp. (a, d)-EAMT) for connected graphs has been extensively investigated. For instances, in [2], Bača et al. showed that wheel W_n has a (a, d)-SEAMT labeling if and only if $d = 1$ and $n \equiv 1 (\mathrm{mod}\, 4)$; Fan F_n has an (a, d)-SEAMT if $2 \leq n \leq 6$ and $d \in \{0, 1, 2\}$. Ngurah and Baskoro [6] proved that for every Petersen graph $P(n, m), n \geq 3, 1 \leq m \leq \frac{n}{2}$, has a $(4n + 2, 1)$-SEAMT labeling. More results concerning SEAMT graphs can be seen in [1] and survey paper by Gallian [4]. People also consider how to construct a new (bigger) (a, d)-SEAMT graphs from some known (smaller) (a, d)-SEAMT graphs. These constructions are proposed by inserting some new pendant edges and points, see for instance [3, 7, 8, 10, 11]. However, the (a, d)-SEAMT labeling for *t*-joint copies of wheel, $W_{(t, n)}$, is still open.

In this paper, we show that $W_{(t, n)}$ has a $((2n + 2)t + 2, 1)$-SEAMT labeling for even $n \geq 4$, and $t \geq 2$.

2. Preliminary Theorems

The properties of (a, d)-SEAMT labeling of graph proposed in the following theorems will be useful in the next section. Given any (a, d)-EAMT labeling λ on a (p, q)-graph G. Then, its dual labeling λ' can be defined [12] by

$$\lambda'(x) = p + q + 1 - \lambda(x) \text{ for any vertex } x, \text{ and}$$

$$\lambda'(xy) = p + q + 1 - \lambda(xy) \text{ for any edge } xy.$$

By using this duality, Wallis et al. [12] established the following theorem.

Theorem A. (Wallis et al. [12]). *If a* (p, q)*-graph G has an* (a, d)*-EAMT labeling, then G has an* (a', d)*-EAMT labeling as its dual with* $a' = 3p + 3q + 3 - a - (q-1)d$.

Theorem B. (Sudarsana et al. [9]). *Let* λ *be a* (a, d)*-SEAMT labeling of a* (p, q)*-graph G. Then the labeling* λ' *defined*:

$$\lambda'(x) = p + 1 - \lambda_1(x), \ \forall x \in V \ \textit{and}$$

$$\lambda'(xy) = 2p + q + 1 - \lambda_1(xy), \ \forall xy \in E$$

is a (a', d)*-SEAMT labeling of G with* $a' = 4p + q + 3 - a - (q-1)d$.

The labeling λ' is called a *dual* (a, d)*-SEAMT labeling* of λ on G. Moreover, the dual labeling is called *selfdual*, if $a = a'$.

3. The Main Result

Let W_n be a wheel on $n+1$ vertices with v_0 as the *hub* and cycle $v_1v_2v_3 \cdots v_nv_1$ as the *rim*. The *t-joint copies of wheel*, denoted by $W_{(t,n)}$, is a graph obtained by taking *t* copies of W_n and joining every hub of the wheels by an edge such that forming a path P_t. We provide an example of $W_{(4,6)}$ in Figure 1:

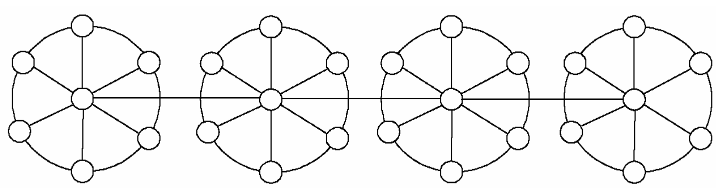

Figure 1. The 4-joint copies of W_6, $W_{(4,6)}$.

In general, we denote that $V(W_{t,n}) = \{v_{i,j} : 1 \le j \le t,\ 0 \le i \le n\}$ and $E(W_{t,n}) = \{e_{i,j} e_{n,j},\ e^o_{i,j},\ e^o_j\}$, where

$$e_{i,j} = \begin{cases} v_{i,j}v_{i+1,j}, & 1 \le i \le n-1, \quad 1 \le j \le t; \\ v_{n,j}v_{1,j}, & i = n, \quad 1 \le j \le t, \end{cases}$$

$$e^o_{i,j} = v_{0,j}v_{i,j}, \quad 1 \le j \le t, \quad 1 \le i \le n,$$

$$e^o_j = v_{0,j}v_{0,j+1}, \quad 1 \le j \le t-1.$$

By (1) and (2), we have: for every even $n \ge 4$, and $t \ge 2$ is an integer, there is no (a, d)-SEAMT labeling of $W_{(t,n)}$ with $d \ge 3$; and there is no (a, d)-EAMT labeling of $W_{(t,n)}$ with $d \ge 5$.

The following theorem deals with $((2n+2)t+2, 1)$-SEAMT labeling of $W_{(t,n)}$.

Theorem 1. *Let* $n \ge 4$ *and* $t \ge 2$ *be integers, and n is even. Then the graph* $W_{(t,n)}$ *has a* $((2n+2)t+2, 1)$*-SEAMT labeling. This type of labeling is selfdual.*

Proof. Label the vertices and edges of $W_{(t,n)}$ in the following way:

$$\lambda(v_{i,j}) = \begin{cases} (i-j)t + j, & 1 \le i \le \frac{n}{2}, \quad 1 \le j \le t; \\ \frac{nt}{2} + j, & i = 0, \quad 1 \le j \le t; \\ it + j, & \frac{(n+2)}{2} \le i \le n, \quad 1 \le j \le t, \end{cases}$$

$$\lambda(e_{i,j}) = \begin{cases} (2n+2-i)t + 1 - j, & 1 \le i \le \frac{n}{2}, \quad 1 \le j \le t; \\ (2n+1-i)t + 1 - j, & \frac{(n+2)}{2} \le i \le n-1, \quad 1 \le j \le t; \\ \left(\frac{3n+2}{2}\right)t + 1 - j, & i = n, \quad 1 \le j \le t, \end{cases}$$

$$\lambda(e^o_{i,j}) = \begin{cases} (3n+3-2i)t+1-j, & 1 \le i \le \frac{n}{2}, \quad 1 \le j \le t; \\ (4n+2-2i)t+1-j, & \frac{(n+2)}{2} \le i \le n, \quad 1 \le j \le t, \end{cases}$$

$$\lambda(e^o_j) = (3n+2)t - j, 1 \le j \le t.$$

Now, we construct the set of edge-weights W in the following way:

$$W = \{w(e_{i,j}), w(e^o_{i,j}), w(e^o_j)\},$$

where

$$w(e_{i,j}) = \begin{cases} \lambda(v_{i,j}) + \lambda(e_{i,j}) + \lambda(v_{i+1,j}), & 1 \le i \le n-1, \ 1 \le j \le t; \\ \lambda(v_{n,j}) + \lambda(e_{n,j}) + \lambda(v_{1,j}), & i = n, \ 1 \le j \le t, \end{cases}$$

$$w(e^o_{i,j}) = \lambda(v_{0,j}) + \lambda(e^o_{i,j}) + \lambda(v_{i,j}), \ 1 \le i \le n, \ 1 \le j \le t;$$

$$w(e^o_j) = \lambda(v_{0,j}) + \lambda(e^o_j) + \lambda(v_{0,j+1}), \ 1 \le j \le t-1.$$

Therefore, we obtain that the edge-weights are

$$w(e_{i,j}) = \begin{cases} (2n+1+i)t+1+j, & 1 \le i \le \frac{n}{2}-1, \quad 1 \le j \le t; \\ \left(\frac{5n+4}{2}\right)t+1+j, & i = \frac{n}{2}, \quad 1 \le j \le t; \\ (2n+2+i)t+1+j, & \frac{(n+2)}{2} \le i \le n-1, \quad 1 \le j \le t; \\ \left(\frac{5n+2}{2}\right)t+1+j, & i = n, \quad 1 \le j \le t, \end{cases}$$

$$w(e^o_{i,j}) = \begin{cases} \left(\frac{7n+5}{2} - i\right)t+1+j, & 1 \le i \le \frac{n}{2}, \quad 1 \le j \le t; \\ \left(\frac{9n+4}{2} - i\right)t+2+j, & \left(\frac{(n+2)}{2}\right) \le i \le n, \quad 1 \le j \le t, \end{cases}$$

$$w(e^o_j) = (4n+2)t+1+j, 1 \le j \le t-1.$$

Finally, we have that the set of edge-weights is

$$W = \{(2n+2)t+2, (2n+2)t+3, ..., (4n+3)t-1, (4n+3)t\}.$$

We can see that the set W consists of the consecutive integers starting from $a = (2n+2)t+2.$ By Theorem B, it can be verified that this labeling is selfdual. The proof is now complete.

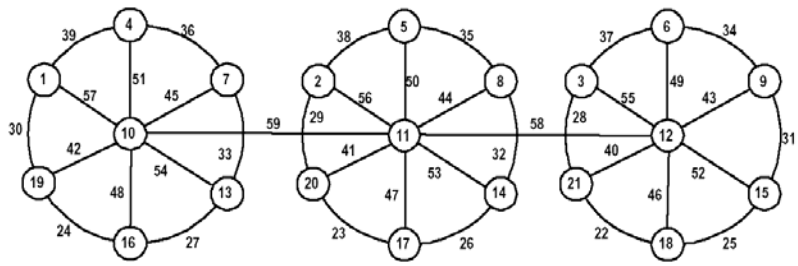

Figure 2. A (44, 1)-SEAMT labeling of $W_{(3,6)}$ obtained from Theorem 1.

By Theorem A, we have the following corollary.

Corollary 1. *For every even* $n \geq 4$ *and* $t \geq 2$ *is an integer, the graph* $W_{(t,n)}$ *has an* $((5n+3)t, 1)$*-EAMT labeling.*

To conclude this paper, let us present two open problems to work on. Construct, if there exists

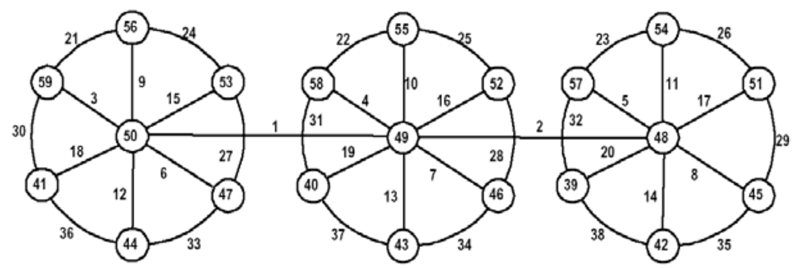

Figure 3. An (99, 1)-EAMT labeling of $W_{(3,6)}$ obtained from Theorems 1 and *A*.

1. An (a, d)-SEAMT labeling of $W_{(t,n)}$ with $d \in \{0, 1\},$ for even $n \geq 4,$ and $t \geq 2.$

2. An (a, d)-SEAMT labeling of $W_{(t,n)}$ with $d \in \{0, 1, 2\}$, for odd $n \geq 5$, and $t \geq 2$.

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