# CONTRIBUTIONS IN MATHEMATICS AND APPLICATIONS III

ICMA-MU 2009, December 2009, Bangkok, Thailand

Editors: Yongwimon Lenbury Nguyen Van Sanh

A special volume 2010 published by East-West Journal of Mathematics

ISSN 1513-489X

### CONTRIBUTIONS IN MATHEMATICS AND APPLICATIONS III

East-West J. of Mathematics, a special volume 2010

Exhausting the circle - on mathematics and history of rectification Yaakov S. Kupitz and Horst Martini	1
Twentieth Century World Mathematics Education Reforms from the Viewpoint of Educational Tools for Enhancing Mathematical Activities Masami Isoda and Maitree Inprasitha	78
Modeling Trust in Open Distributed Multiagent Systems  Tran Dinh Que and Nguyen Manh Hung	98
Planar soap bubbles on a half plane for three and four areas with equal pressure regions  C. Maneesawarng , B. Sroysang, S. Talwong and W. Wichiramala	109
Learning the value of a function from inaccurate data  K. Khompurngson, B. Novaprateep and Y. Lenbury	128
Existence and uniqueness of a blow-up solution for a parabolic problem with a <i>localized nonlinear</i> term via semigroup theory P. Sawangtong , C. Licht, B. Novaprateep and S. Orankitjaroen	139
A Remark on the Homogenization of a Microfibered Linearly Elastic Material N. Sontichai, S. Orankitjaroen, C. Licht and A. Kananthai	153
Residue (at Zero) of Functions Related to Ones Generating the Generalized Bernoulli Polynomials  Aram Tangboonduangjit	166
Linearization of First Order Stochastic Differential Equations  Sergey Meleshko and Eckart Schulz	178
Simulations of Inventory Systems Subject to Different Strategies under Uncertain Demand and Lead-time  C. Rattanakul , W. Sarika , Y. Lenbury , and N. Tumrasvin	187

A classification of some graphs containing wheels based on $f$ -colorings Adiwijaya, A.N.M. Salman, D. Suprijanto and E.T. Baskoro	200
Mathematical Modelling and Numerical Simulation of a Fullerene in a Single-walled Carbon Nanotube K. Srikhaltai, K. Chayantrakom and D. Baowan	208
The Control Design of Symmetric System for Tracking a Desired Path with an Obstacle Using Tracking Error Dynamics  Miswanto, I. Pranoto, H. Muhammad and D. Mahayana	221
2-D Solutions in Re-Entry Aerodynamics  Gabriel Mititelu and Yupaporn Areepong	231
Explicit Analytical Solutions for the Average Run Length of CUSUM and EWMA Charts G. Mititelu, Y. Areepong, S. Sukparungsee and A. Novikov	253
A Variational Approach for Discontinuity-Preserving Image Registration Noppadol Chumchob and Ke Chen	266
Some Cycle-(Super)Magic Labelings of Some Complete Bipartite Graphs A.N.M. Salman and A.D. Purnama	283
Curve Shortening on Sasaki Manifolds and the Weinstein Conjecture  Knut Smoczyk	292
The Tarry-Escott problem of degree two over quadratic fields  S. Prugsapitak	306
Sinc-Galerkin Method for the Option Pricing Under Jump-Diffusion Model  Jun Liu and Hai-Wei Sun	317
Using Singularity Theory to Analyse a Spatially Uniform Model of Self-Heating in Compost Piles  T. Luangwilai, H.S. Sidhu, M.I. Nelson and X.D. Chen	328

)

#### Contribution in Mathematics and Applications III East-West J. of Mathematics, a special volume 2010, pp. 200-207

## A CLASSIFICATION OF SOME GRAPHS CONTAINING WHEELS BASED ON f-COLORINGS

Adiwijaya\*, A.N.M. Salman D. Suprijanto and E.T. Baskoro

Combinatorial Mathematics Research Group, Fakultas Matematika dan Ilmu Pengetahuan Alam, Institut Teknologi Bandung, Jl. Ganesa 10 Bandung 40132, Indonesia e-mail: adiwijaya007@yahoo.co.id

#### Abstract

Let G=(V(G),E(G)) be a simple graph and f be a function from V to a subset of positive integers. An f-coloring of G is a generalized edge-coloring such that every vertex  $v \in V$  has at most f(v) edges colored with a same color. The minimum number of colors needed to define an f-coloring of G is called an f-chromatic index of G, denoted by  $\chi'_f(G)$ . A problem in the f-coloring is how to determine  $\chi'_f(G)$  of a given graph G. Based on the f-chromatic index, a graph G can be either in the  $C_f$ 1 or  $C_f$ 2. In this paper, we consider a graph containing wheels, especially the corona product of either the complement of a complete graph, or a path, or a star with a cycle. We give a classification of these graphs based on f-colorings.

#### 1. Introduction

In this paper, we deal with simple graphs which are finite undirected graphs without loops or multiple edges. Let G = (V(G), E(G)) be a graph with the

**Keywords:** corona product, f-chromatic index, f-coloring. 2000 AMS Mathematics Subject Classification: 05C15

<sup>\*</sup>Permanent Address: Faculty of Science Institut Teknologi Telkom, Jl. Telekomunikasi no. 1 Bandung 40257, Indonesia.

where set V(G) and the edge set E(G). Let f be a function from V(G) to a subset of positive integer. An f-coloring of G is a coloring of edges such that and vertex v has at most f(v) edges colored with a same color. The minimum limber of colors needed to define an f-coloring of G is called an f-chromatic water of G, denoted by  $\chi'_f(G)$ .

A problem in the f-coloring is how to determine  $\chi'_f(G)$  of a given graph G. Farses in many applications, including network design problem, the scheduling problems, and file transfer problems in a computer network [4]. The file transfer problem in a computer network is modeled as follows. Each computer is represented by a vertex and every file transfer process between two computers is represented by an edge. Each computer v has a limit number f(v) of communication ports. If we assume that the transfer time is constant for every file, we can use an f-coloring to manage transferring all files along the minimum time needed.

Let.

$$\Delta_f(G) = \max_{v \in V(G)} \left\{ \left\lceil \frac{d(v)}{f(v)} \right\rceil \right\},\tag{1}$$

where d(v) is the degree of v. Hakimi and Kariv [5] showed that

$$\Delta_f(G) \le \chi_f'(G) \le \Delta_f(G) + 1. \tag{2}$$

G is called in the class-1, denoted by  $G \in C_f 1$ , if  $\chi'_f(G) = \Delta_f(G)$ ; otherwise G is called in the class-2, denoted by  $G \in C_f 2$ . Holyer [6] proved that the edgewloring problem is an NP-complete. It is reduced from the 3SAT problem. Consequently, the f-coloring problem is an NP-complete problem.

Hakimi and Kariv [5] showed that any bipartite graph is in  $C_f1$ . In [2], we showed that any helm graphs, any gear graphs and some friendship graphs are in  $C_f1$ . If G is a graph with even f(v) for each  $v \in V(G)$ , then G is in  $C_f1$ . Yu et al. [8] gave sufficient conditions for fans and wheels to be in  $C_f1$ . Thang and Liu [10] found the f-chromatic index for complete graphs and gave a classification of complete graphs based on f-colorings. In 2008, Zhang et al. [11] presented a classification of regular graphs based on f-coloring.

$$V_0^*(G) = \left\{ v \mid \frac{d(v)}{f(v)} = \Delta_f(G), v \in V(G) \right\},\tag{3}$$

and

$$V^*(G) = \left\{ v \mid \left\lceil \frac{d(v)}{f(v)} \right\rceil = \Delta_f(G), v \in V(G) \right\}.$$
 (4)

Thang and Liu in [9] gave some sufficient conditions for a graph to be in  $C_f 1$  sollows.

**Theorem 1.1** [9] Let G be a graph. If the subgraph induced by  $V_0^*(G)$  is forest, then  $G \in C_f 1$ .

**Theorem 1.2** [9] Let G be a graph. If  $d(v^*)$  is not devided by  $f(v^*)$  for every  $v^* \in V^*(G)$ , then  $G \in C_f 1$ .

We use [a,b] instead of  $\{x \in \mathbb{N} | a \leq x \leq b\}$ . Let G and H be two graphs with n and m vertices, respectively. The corona product of G with H, denoted by  $G \odot H$ , is a graph obtained by taking one copy of G and n copies of H, namely  $H_1, H_2, \ldots, H_n$ , and then for  $i \in [1,n]$ , joining the i-th vertex of G to every vertex of  $H_i$ . Here G is called the center of  $G \odot H$  and H is called the outer of  $G \odot H$ . In this paper, a vertex set and an edge set in the enter of  $G \odot H$  is denoted by V(G) and E(G), respectively. A vertex set and an edge set in the outer of  $G \odot H$  is denoted by V(H) and E(H), respectively. We know that the corona product of any graph with a cycle produces a graph containing wheels. We have shown that the corona product of a cycle with either the complement of a complete graph, or a path, or a cycle is in  $C_f 1$  and the corona product of a complete graph with a cycle is in  $C_f 1$  and the corona product of a complete graph with a cycle is in  $C_f 1$  and

In this paper, we consider some other graphs containing wheels, namely the corona product of either the complement of a complete graph, or a path, or a star with a cycle. In Theorem 2.1, we give a classification of the corona product of the complement of a complete graph with a cycle. In [3], we have shown that the corona product of a cycle with a path is in  $C_f1$ . It is well-known that the corona product of any two graphs is not commutative. Hence, it is natural to look for a sufficient condition of the corona product of a path with a cycle to be in  $C_f1$ . In Theorem 2.2, we give a classification of the corona product of a path with a cycle. Moreover, in Theorem 2.3, we give a classification of the corona product of a star with a cycle.

#### 2. Main Results

In this paper, we associate positive integers with colors. Let F be an f-coloring of G. Let  $F^{-1}(i)$  denotes the set of edges of G that receive color i under F and  $F_v^{-1}(i)$  denote the set of edges of G which is incident with the vertex v and receive color i under F.

Let  $W_m=K_1\odot C_m$  be a wheel on m+1 vertices. Let  $E_0=E(C_m)$  and  $E_1$  be a set of edges which are incident with  $V(K_1)$ . Let C be an f-coloring of  $W_m$  such that for  $i\in [1,k], |C_v^{-1}(i)\cap E_0|=1$  for every  $v\in V(C_m)$  (except when k=2 and  $|E_0|$  is odd, there is one and only one vertex  $v\in V(C_m)$  with  $|C_v^{-1}(i)\cap E_0|=2)$  and  $a_1\geq a_2\geq \ldots \geq a_k$  where  $a_i=|C^{-1}(i)\cap E_0|$ . Let  $b_i=|C^{-1}(i)\cap E_1|$ , we have two following conditions:

- 1. If m is even and  $a_i = \frac{m}{2} t_i$  for some  $t_i \in [0, (\frac{m}{2} 1)]$ , then  $b_i \leq 2t_i$ ,
- 2. If m is odd and  $a_i = \lfloor \frac{m}{2} \rfloor t_i$  for some  $t_i \in [0, (\lfloor \frac{m}{2} \rfloor 1)]$ , then  $b_i \leq 2t_i + 1$ .

We will use the f-coloring C to prove Theorem 2.1, Theorem 2.3, and Theorem 2.3.

In the Theorem 2.1, we give a classification of the corona product of the complement of a complete graph  $(K_n^c)$  with a cycle based on f-colorings.

Theorem 2.1 Let 
$$n \geq 1$$
,  $m \geq 3$  and  $G = K_n^c \odot C_m$ .

If either  $m = 5$  or  $m = 8$  and  $f(v) = \begin{cases} 1, & v \in V(C_m), \\ \lceil \frac{m}{3} \rceil, & v \in V(K_n^c), \end{cases}$  then  $G \in C_f 2$ . Otherwise,  $G \in C_f 1$ .

Proof.

Let n=1. If f fulfills the premise of the theorem, then  $\Delta_f(G)=3$ . If  $\Delta_f(G)=1$ , it is clear that we can construct an f-coloring of G with one color. For  $\Delta_f(G)\geq 2$ , we divide the proof into three cases as follows.

Case 1. 
$$\Delta_f(G) = 2$$

We can color all edges in  $E_0$  and  $E_1$  such that  $|C_v^{-1}(i)| \leq 2$  for every  $v \in V(C_m)$  and  $|C_w^{-1}(i)| \leq \lceil \frac{m}{2} \rceil$  for the vertex  $w \in V(K_1)$ . Hence,  $G \in C_f 1$ .

Case 2. 
$$\Delta_f(G) = 3$$

In this case, we divide the proof into three subcases.

Subcase 2.1  $m \neq 5$  and  $m \neq 8$ . Let m = 3r + s for some  $s \in [0, 2]$  and  $r \in \mathbb{N}$ . We can construct an f-coloring C by using 3 colors such that  $|C_v^{-1}(i)| = 1$  for every vertex  $v \in V(C_m)$  and  $|C_w^{-1}(i)| \leq r + 1$  for the vertex  $w \in V(K_1)$ . Hence,  $G \in C_f 1$ .

Subcase 2.2 For either m=5 or m=8 and f fulfills the premise of the theorem. We assume that there exists an f-coloring C of G by using 3 colors, we have the conditions as follows.

For m=5, let  $h_1, h_2, h_3$  be a monotone non-increasing sequence of the number of edges in  $E_0$  with a same color such that  $h_1 \geq h_2 \geq h_3$ . Let  $t_i \in [0,1]$ , since  $h_1+h_2+h_3=5$ , we get  $t_1+t_2=0$ . Let  $g_1, g_2, g_3$  be a number of edges in  $E_1$  with a same color, respectively. Since  $g_1+g_2+g_3=5$ , we get  $g_3 \geq 3$ . It means that f(w) must be 3 for  $w \in V(K_1)$ . We get a contradiction. It is impossible to construct an f-coloring by using 3 colors. Hence,  $G \in C_f 2$ .

For m=8, let  $t_i \in [0,3]$ . By using a similar technique, we have  $h_1+h_2+h_3=8$ . It implies  $t_1+t_2=h_3$ . Since  $g_1+g_2+g_3=8$ , we get  $2h_3+g_3\geq 8$  where  $h_3\leq 2$ . It implies  $h_3\geq 4$ . It means that f(w) must be 4 for  $w\in V(K_1)$ .

We get a contradiction. It is impossible to construct an f-coloring by using a colors. Hence,  $G \in C_f 2$ .

Subcase 2.3 For either m=5 or m=8 and there exists  $u\in V(C_m)$  with  $f(u)\neq 1$  or  $w\in V(K_1)$  with  $f(w)\neq \lceil \frac{m}{3}\rceil$ , we can color all edges of G by using the similar technique in Subcase 2.1 by using 3 colors such that  $C_u^{-1}(i)\neq 1$  or  $C_w^{-1}(i)\neq \lceil \frac{m}{3}\rceil$  for some  $i\in [1,3], u\in V(C_m)$  and  $w\in V(K_1)$ . Hence,  $G\in \mathcal{C}_{f^{\perp}}$ 

Case 3.  $\Delta_f(G) \geq 4$ 

We obtain  $V^* \subseteq V(K_1)$ . If  $V_0^* = V(K_1)$ , by Theorem 1.1, we have  $G \in C_{f^{\perp}}$ . Otherwise, by Theorem 1.2, we have  $G \in C_{f^{\perp}}$ .

Let  $n \geq 2$ . Since,  $K_n^c \odot C_m$  is the *n*-copies of  $K_1 \odot C_m$ , we have a conclusion that if f fulfills the premise of the theorem, then  $G \in C_f 2$ . Otherwise,  $G \in C_f 1$ .

Let  $G = P_n \odot C_m$  be the corona product of a path on n vertices with a cycle on m vertices. Thus, we have the following theorem:

**Theorem 2.2** Let 
$$n \geq 2$$
,  $m \geq 3$ , and  $G = P_n \odot C_m$ .  
 If  $m = 5$  and  $f(v) = \begin{cases} 1, & \text{for } v \in V(C_m), \\ \lceil \frac{m}{3} \rceil, & \text{for } v \in V(P_n) \text{ and } d(v) = m+1, \end{cases}$  then  $G \in C_f 2$ . Otherwise,  $G \in C_f 1$ .

#### Proof.

We can color all edges of wheels by using the similar technique for every case in Theorem 2.1. Next, we color every edge of the path in three following cases.

Case  $1 \Delta_f(G) = 2$ 

We color all edges of the path by 1. Hence,  $G \in C_f 1$ .

Case 2  $\Delta_f(G) = 3$ 

We divide the proof into two subcases.

Subcase 1 For m=5 and f fulfills the premise of the theorem, by using the same reason in the proof of Theorem 2.1 subcase 2.2, we can not construct an f-coloring by using 3 colors. Hence,  $G \in C_f 2$ .

Subcase 2 For  $m \neq 5$  or f(v) does not fulfill the premise of the theorem, we color all edges of the path by 1,2 alternately. But, when m=4 or m=7, we color all edges of the path by 1. Hence,  $G \in C_f 1$ .

Case 3  $\Delta_f(G) \geq 4$ 

We obtain  $V^* \subseteq V(P_n)$ . If there exists  $v \in V(P_n)$  such that  $v \in V_0^*$ , by Theorem 1.1, we have  $G \in C_f 1$ . Otherwise, by Theorem 1.2, we have  $G \in C_f 1$ .

Let  $S_n$  be a star on n+1 vertices. In Theorem 2.3, we give a classification of the corona product of a star with a cycle based on f-colorings.

Theorem 2.3 Let  $n \geq 3$  and  $m \geq 3$ ,  $G = S_n \odot C_m$ . If m = 5 and  $f(v) = \begin{cases} 1, & v \in V(C_m), \\ \lceil \frac{m}{3} \rceil, & v \text{ are } n \text{ pendant vertices in } V(S_n), \end{cases}$  then  $G \in C_f 2$ . Otherwise,  $G \in C_f 1$ .

Proof.

Suppose, we can construct an f-coloring C by using  $\Delta_f(G)$  colors. Let the center of the star be labeled by w and are pendant vertices of the star be labeled by  $v_1, v_2, ..., v_n$ , respectively. If f fulfills the premise of the theorem, then  $\Delta_f(G) = 3$ . If  $\Delta_f(G) = 1$ , it is clear that an f-coloring of G with one color. For  $\Delta_f(G) \geq 2$ , we divide the proof into three cases as follows.

Case 1.  $\Delta_f(G) = 2$ 

For  $i \in [1, \lceil \frac{m}{2} \rceil]$ , we color all edges of wheels which the center of wheel is  $v_{i-1}$  by using f-coloring C such that  $|C_v^{-1}(i)| \leq 2$  for every  $v \in V(C_m)$  and  $|C_{v_{2i-1}}^{-1}(i)| \leq \lceil \frac{m}{2} \rceil$ . If the center of wheel is  $v_{2i}$ , we color all edges of wheels by using the similar technique but we have to replace color 1 with 2 and otherwise. Finally, for  $i \in [1, n]$ , we color  $wv_i$  by 1, 2 alternately and we color the wheel which the center of wheel is w by using f-coloring f. Hence, f is f in f i

Case 2.  $\Delta_f(G) = 3$ 

We divide the proof into three subcases.

Subcase 2.1 For  $m \neq 5$ . Let m = 3r + s for some  $s \in [0, 2]$  and  $r \in [1, \lceil \frac{m}{3} \rceil)$ . We can color all edges of wheels as follows.

- 1. If the center of the wheel is  $v_{3r-2}$ , then we construct an f-coloring C by using the similar technique in the Theorem 2.1 Subcase 2.1.
- 2. If the center of the wheel is  $v_{3r-1}$ , then we construct an f-coloring C by using the similar technique in the Theorem 2.1 Subcase 2.1, but we have to replace color 1 with 2 and otherwise.
- 3. If the center of the wheel is  $v_{3r}$ , then we construct an f-coloring C by using the similar technique in the Theorem 2.1 Subcase 2.1, but we have to replace color 1 with 3 and otherwise.

Finally, for all edges of the wheel which the center of the wheel is w, we construct an f-coloring by using the similar technique in the Theorem 2.1 Subcase 2.1 and for  $j \in [1, m]$ , we color  $wv_j$  by 1, 2, 3, alternately. Hence,  $G \in C_f$ 1.

Subcase 2.2 For m=5 and f fulfills premise of the theorem, by using the same reason in the proof of the Theorem 2.1 Subcase 2.2, we can not construct an f-coloring by using 3 colors. Hence,  $G \in C_f 2$ .

Subcase 2.3 For m=5 and there exists  $u\in V(C_m)$  with  $f(u)\neq 1$  or  $w\in V(K_1)$  with  $f(w)\neq \lceil\frac{m}{3}\rceil$ , we can color all edges of G by using the similar technique in subcase 2.1 by using 3 colors such that  $C_u^{-1}(i)\neq 1$  or  $C_v^{-1}(i)\neq \lceil\frac{m}{3}\rceil$  for some  $i\in [1,3],\ u\in V(C_m)$  or  $v\in \{v_1,v_2,...,v_n\}$ . Hence,  $G\in C_f1$ .

Case 3.  $\Delta_f(G) \geq 4$ 

We obtain  $V^* \subseteq V(S_n)$ . If there exist  $v \in V(S_n)$  such that  $v \in V_0^*$ , by Theorem 1.1, we have  $G \in C_f$ 1. Otherwise, by Theorem 1.2, we have  $G \in C_f$ 1.

Acknowledgements The Authors would also like to thank Prof. Oriol Serra Universitat Politècnica de Catalunya, Barcelona, Spain) for many helpful suggestions. The Authors would also like to thank Directorate General of Higher Education (DGHE) Department of National Education, Indonesia, for financial supporting.

#### References

- [1] Adiwijaya, A.N.M. Salman, E.T. Baskoro, D. Suprijanto, On the f-coloring of the corona product of  $K_n$  with  $C_n$ , the Proceeding of International Conference on Mathematics and Statistics (2008) 298 301.
- [2] Adiwijaya, A.N.M. Salman, D. Suprijanto, E.T. Baskoro, f-chromatic indexes of graphs like-wheel, Proceeding of International Conference on Mathematics, Statistics, and its Applications (2009) 24-27
- [3] Adiwijaya, A.N.M. Salman, D. Suprijanto, E.T. Baskoro, On the f-colorings of the corona product of a cycle with some graphs, Journal of Combinatorial Mathematics and Combinatorial Computation 71 (2009) 235-241.
- [4] E. G. Coffman, M.R. Garey, D.S. Johnson, A.S. LaPaugh, Scheduling file transfers, SIAM Journal of Computation 14:3 (1985) 744-780.
- [5] S.L. Hakimi, O. Kariv, A generalization of edge-coloring in graphs, Journal of Graph Theory 10 (1986) 139-154.
- [6] I. Holyer, The NP-completness of edge-coloring, SIAM Journal of Computation 10:4 (1981) 718 720.
- [7] S. Nakano, T. Nishizeki, N. Saito, On the f-coloring of multigraphs, IEEE Transaction Circuit System 35:3 (1988) 345-353.
- [8] J. Yu, L. Han, G. Liu, Some result on the classification for f-colored graphs, Proceeding ISORA (2006) 292-298.
- $\P$  X. Zhang, G. Liu, Some sufficient conditions for a graf to be  $C_f$ 1, Applied Mathematics Letters 19 (2006) 38-44.
- [0] X. Zhang, G. Liu, The classification of  $K_n$  on f-colorings, Journal of Applied Mathematics and Computing 19:1-2 (2006) 127-133.
- [II] X. Zhang, J. Wang, G. Liu, The classification of regular graphs on f-colorings, Ars Combinatoria 86 (2008) 273-280.

Contributions in Mathematics and Applications III

In Inter. Conference in Mathematics and Applications, ICMA-MU 2009, Bangkok. Oppright ©by East-West J. of Mathematics.

Might of reproduction in any form reserved.